

# On Ionization Equilibrium of Helium

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The ionization equilibrium of helium is investigated using new ionization and modified dielectric recombination rate coefficients. The results are discussed in the light of available data.

## 1. Introduction

The emission lines of an ion ( $X^{+m}$ ) are detectable only from those regions of the solar corona where the temperature is close to  $T_{\max}(X^{+m})$ , the temperature at which the ion  $X^{+m}$  has maximum concentration. A knowledge of the ionization equilibrium which provides information regarding  $T_{\max}(X^{+m})$  is therefore of considerable interest. Extensive calculations of the ionization equilibrium for a large number of elements and for a wide range of temperatures have been carried out by House<sup>1</sup>, Tucker and Gould<sup>2</sup>, Allen and Dupree<sup>3</sup>, Jordan<sup>4</sup>, Ansari et al.<sup>5</sup> and Landini and Monsignori Fossi<sup>15</sup>. Most of them, however, have used the collisional ionization rate coefficients of Seaton<sup>7</sup> in their investigations.

Recently Narain and Chandra<sup>8</sup> and Chandra<sup>9</sup> have shown that Seaton's ionization formula is a special case of that of Lotz<sup>10</sup> in the low energy region. Also Lotz's formula supports the observations made by Burgess<sup>11</sup> (Chandra and Narain<sup>12</sup>).

Burgess<sup>13</sup> showed that in the coronal temperature range the dielectronic recombination was more important than the radiative one. He proposed a general formula (Burgess<sup>14</sup>) for the estimation of dielectronic recombination coefficients. Since then, a number of investigations have been carried out in this direction, the most extensive one being that of Ansari et al.<sup>5</sup>. Landini and Monsignori Fossi<sup>15</sup> presented two simple expressions for the total dielectronic recombination coefficient. Their expression for recombining ions pertaining to H, He, Ne, K–Ni sequences, is expected to overestimate the dielectronic coefficient.

Here we have used the ionization rate coefficients given by Lotz<sup>10,16</sup> and a modified version of the

expression of Landini and Monsignori Fossi<sup>15</sup> to investigate the ionization equilibrium of helium.

## 2. Theoretical Details

Elwert<sup>17</sup> has shown that the processes of photoionization and the three body recombination are not significant in the solar corona. The collisional ionization from the ground terms, the collisional excitation followed by autoionization, radiative recombination via the continuum and the bound levels and the dielectronic recombination reduced by a density dependent term are the main processes responsible for setting up an equilibrium. In this context the ratio of the concentrations of the two successive stages of ionization may be written as

$$\frac{N(X^{+m+1})}{N(X^{+m})} = \frac{q_{\text{coll}} + q_{\text{aut}}}{\alpha_{\text{rad.c}} + \alpha_{\text{rad.b.l.}} + \alpha_{\text{d}}(\text{eff})} \quad (2.1)$$

where the symbols have their usual meanings (Jordan<sup>4</sup>). The ionization equilibrium  $N(X^{+m})/N(X)$ , where  $N(X)$  is the total number density of the element ( $X$ ), is calculated from the values of  $N(X^{+m+1})/N(X^{+m})$ .

Following Lotz<sup>10</sup> the collisional ionization rate is given by

$$q_{\text{coll}} = 6.7 \cdot 10^{-7} \sum_{i=1}^N \frac{a_i q_i}{(kT)^{3/2}} \left[ \left( \frac{P_i}{kT} \right)^{-1} \int_{P_i/kT}^{\infty} \frac{e^{-x}}{x} dx - \frac{b_i \exp(c_i)}{(P_i/kT) + c_i} \int_{(P_i/kT) + c_i}^{\infty} \frac{e^{-y}}{y} dy \right] \text{cm}^3 \text{sec}^{-1} \quad (2.2)$$

where  $a_i$  is given in  $10^{-14} \text{cm}^2 (\text{eV})^2$  and  $P_i$  and  $kT$  in eV. The symbols have their usual meaning (Lotz<sup>10</sup>). For the case of He and  $\text{He}^+$ ,  $N=1$ . The values of other parameters used in the calculation of (2.2) are listed in Table 1. The autoionization rate coefficient,  $q_{\text{aut}}$ , may be calculated by the formula given by Seaton<sup>7</sup>. Since the ground terms of He and  $\text{He}^+$  have no inner shell,  $q_{\text{aut}}$  is, there-

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Table I

Species	Conf.	$q_1$	$P_1$ (eV)	$a_1^*$	$b_1$	$c_1$
He	1 s <sup>2</sup>	2	24.6	4.0	0.75	0.46
He <sup>+</sup>	1 s <sup>1</sup>	1	54.4	4.4	0.38	0.60

\* in  $10^{-14} \text{ cm}^2 (\text{eV})^2$ .

fore, neglected in the present investigation (Jordan<sup>4</sup>).

For temperatures  $T \leq 6 \cdot 10^5 \text{ K}$ , the radiative recombination rate via continuum has been calculated using the well known formula of Elwert<sup>18</sup> given by

$$\alpha_{\text{rad,c}}(X^{+m}) = 0.97 \cdot 10^{-12} I_m n_0 g T^{-1/2} \text{ cm}^3 \text{ sec}^{-1} \quad (2.3)$$

where the symbols have their usual meanings (Jordan<sup>4</sup>). For higher temperatures the formula given by Burgess and Seaton<sup>19</sup> has been applied.

The dielectronic recombination coefficients are calculated using the expression (Landini and Monsignor Fossi<sup>15</sup>)

$$\alpha_{\text{diel}}(X^{+m}) = 2 \cdot 10^{-4} T^{-3/2} (m+2)^2 f_{1,0} W_1^{1/2} \exp \{ -10.6 \cdot 10^3 W_1/T \} \text{ cm}^3 \text{ sec}^{-1} \quad (2.4)$$

where  $W_1$  is the excitation potential (in eV) of the first allowed level of the recombining ion, and

$$f_{1,0} = \sum_j f_j = n, \quad (2.5)$$

$n$  being the number of electrons in the outer shell of the recombining ion. Since  $\sum_j f_j$  includes the contributions of discrete as well as those of continuum states, the coefficient would, therefore, be overestimated. For example, for the process  $\text{He} \leftarrow \text{He}^+$ , it is erroneous to take  $f_{1,0} = 1$ . Similarly for other processes the  $f_{1,0}$  should be reduced appropriately. In the present investigation we have taken  $f_{1,0} = 0.5641$  (Allen<sup>20</sup>) which represents the contribution of discrete states only.

The coefficients obtained using (2.4) and the above mentioned value of  $f_{1,0}$  are corrected for density dependence. This is necessary because the levels higher than the thermal limit  $n_t$ , given by (Wilson<sup>21</sup>)

$$n_t^7 = 14 \cdot 10^{14} (m+1)^6 T^{1/2} N e^{-1} \quad (2.6)$$

do not contribute to the dielectronic recombination. The values of the reduction factor needed in these calculations, following Jordan<sup>4</sup>, are taken from Burgess<sup>22</sup>.

The radiative recombination coefficients via bound levels are calculated using the formula (Wilson<sup>21</sup>)

$$\alpha_{\text{rad,b,l}}(X^{+m}) = 1.2 \cdot 10^{-6} (m+1)^4 T^{-3/2} n_t^{-1} \exp \{ -X_t/kT \} \text{ cm}^3 \text{ sec}^{-1}, \quad (2.7)$$

in which  $X_t$  is given by

$$X_t^7 = 6 \cdot 10^{-28} I_m N e^2/kT. \quad (2.8)$$

### 3. Results and Discussion

In order to show that the ionization rate coefficients used by us differ from those of Seaton<sup>7</sup>, we have exhibited them in Figure 1. Figure 2 displays

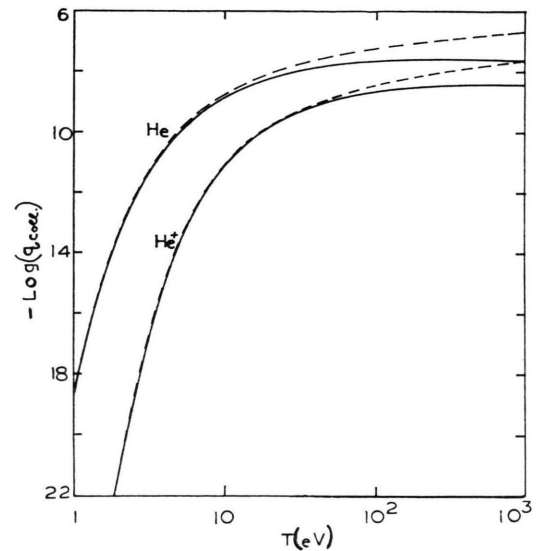


Fig. 1. Coefficients of the collisional ionization rate for He and He<sup>+</sup>: — using the formula of Lotz<sup>10</sup>; --- using the formula of Seaton<sup>7</sup>.

the dielectronic recombination coefficients of Burgess<sup>13</sup> and those obtained using the formula of Shore<sup>23</sup> along with the present values. In Fig. 3 the present values of  $\log[N(X^{+m+1})/N(X)]$  are plotted as a function of temperature along with those of House<sup>1</sup>.

It is obvious from Fig. 1 that the present ionization rate coefficients for He differ from those of Seaton<sup>7</sup> above 5 eV while those for He<sup>+</sup> differ above 50 eV. Therefore, the present results for ionization equilibrium should differ slightly from those calculated using Seaton's formula.

Ansari et al.<sup>5</sup> have indicated that the dielectronic recombination coefficients using the formula of

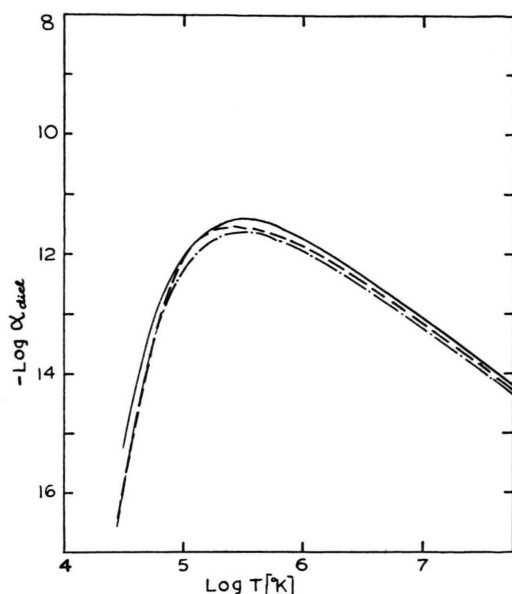


Fig. 2. Coefficients of dielectronic recombination for the process  $\text{He} \leftarrow \text{He}^+$ . — present calculations; --- results of Burgess<sup>13</sup>; - · - · - results obtained using the formula of Shore<sup>3</sup>.

Tucker and Gould<sup>2</sup> would overestimate the coefficients obtained using Burgess's formula by a factor of 3 to 8. The coefficients obtained from (2.4) with  $f_{1,0} = 1$  are also larger by a factor of about 1.8 than those of Burgess<sup>13</sup> whereas those obtained using  $f_{1,0} = 0.5641$  agree nicely. Therefore, the coefficients used by us would produce a significant change in the ionization equilibrium results as compared to those of Tucker and Gould.

The ionization equilibrium results exhibited in Fig. 3 indicate that the present results differ significantly from those of House<sup>1</sup> who did not include dielectronic recombination. The results of House<sup>1</sup> show that the maximum abundance of  $\text{He}^+$  will lie in the temperature range of 0.6 to 0.675 eV. The present study leads to the range 0.55 to 0.70 eV. The inclusion of dielectronic recombination, therefore, broadens the peak for the maximum abundance of  $\text{He}^+$ . Similar observations have been made by Tucker and Gould<sup>2</sup>. But their results are erroneous because of the drastic assumptions made in the esti-

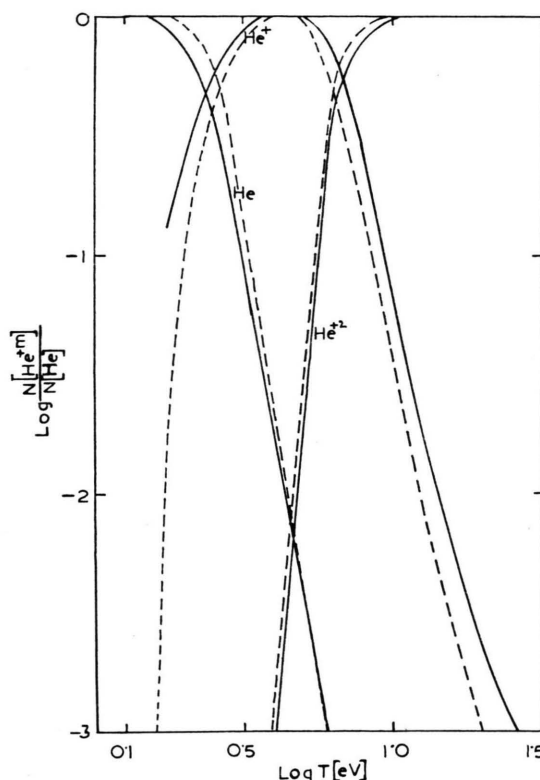


Fig. 3. Ionization Equilibrium of He-ions. — present calculations; --- results of House<sup>1</sup>.

mation of dielectronic coefficients (Ansari et al.<sup>5</sup>).

It may be concluded that the ionization rate coefficients of Lotz<sup>10</sup> should be preferred at higher temperatures. The dielectronic coefficients for H, He, Ne, K – Ni sequences should be estimated by (2.4) only after proper reduction of  $f_{1,0}$ . The dielectronic recombination broadens the peak for the maximum abundance in agreement with Jordan<sup>4</sup>.

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